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Aerodynamic Mechanisms of Nonlinear Flutter with Subcritical Hopf Bifurcation of a Bridge Girder

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SUMMARY:
This study investigates the nonlinear aerodynamic properties and actual energy feedback mechanism of limit cycle oscillation (LCO) and subcritical Hopf bifurcation of a truss girder. The response characteristics of the girder were experimentally investigated through section model wind tunnel tests. A modified nonlinear self-excited force model was proposed to investigate the intrinsic time-varying characteristics of the aerodynamic properties and the real energy feedback mechanism of the subcritical Hopf bifurcation. Based on the model, the characteristics of nonlinear self-excited forces and their contribution to the energy exchange behaviors, were studied. Subsequently, the energy feedback mechanisms of LCO and subcritical Hopf bifurcation were qualitatively discussed in detail considering the hysteresis loop of nonlinear self-excited forces, which highlighted the important role of the linear self-excited-moment in the generation of a subcritical Hopf bifurcation. Finally, the evaluation of the existence of subcritical Hopf bifurcation was intuitively reflected in the aerodynamic parameters of the nonlinear self-excited force model, which can help attain better bridge deck tailoring to achieve a better aerodynamic configuration.

Keywords: Truss girder, Subcritical Hopf bifurcation, Nonlinear self-excited forces, Limit cycle oscillation.

1. INTRODUCTION

The flutter instability of bridges is a self-excited aerodynamic instability. It has attracted considerable attention over the past several decades. According to certain well-developed linear methodologies, the critical wind speed, as well as the dynamic mechanism of flutter can be accurately predicted and interpreted (e.g., Chen et al. 2000; Miyata, 2003;). However, these methodologies often assume linearized self-excited forces in terms of linear flutter derivatives. Consequently, these methodologies are only applicable to the calculation of linear flutter.

However, experiences revealed that aerodynamic nonlinearity becomes gradually prominent in wind-induced motions. Instead of a linear flutter, a nonlinear one with Hopf bifurcation was generally observed (e.g., Amandolese et al., 2013; Wu et al., 2020; Gao et al., 2020). The Hopf bifurcation be classified into two groups by judging the stability of the LCO, that is, the supercritical one when all LCOs are stable and the subcritical one when unstable LCOs exists. Great efforts have been paid to developing analysis frameworks for investigating the supercritical Hopf bifurcation of bridge decks. However, less attentions are paid to the calculation and driving mechanism of the subcritical Hopf bifurcation of bridges.

In this study, the subcritical Hopf bifurcation of a truss girder was examined through a wind tunnel test. The characteristics of torsional flutter and vertical motion on the evolution of unstable LCO are analyzed. An empirical nonlinear self-excited force model was proposed. The hysteresis loops of nonlinear self-excited forces are generated by which the aerodynamic mechanism of LCO and subcritical Hopf bifurcation can be well explained.

2. SUBCRITICAL HOPF BIFURCATION OF A TRUSS GIRDER

A truss girder, as shown in Figure 1, is adopted. The section model was modeled with a scale ratio of 1:80. Which has a length of 1.1m, a width of 0.35m and a height of 0.125m. Two degrees-of-freedom system (2DOFs) was built to allow vibration in both the vertical and torsional directions. Besides, an SDOF system was also built to simulate torsional vibration only. The flow condition was smooth flow. Further, the wind angle of attack (AOA) was set at 0º. The mass and mass moment of inertia per unit length of both systems were 6.36 *kg*/*m* and 0.123 $kg{\cdot}m^2/m$, respectively. The natural frequencies in the torsional and vertical directions, were f_α=3.37Hz and f_h=*2.76*Hz, respectively. Detailed information regarding the experimental setup can be found in Wu et al. (2020).

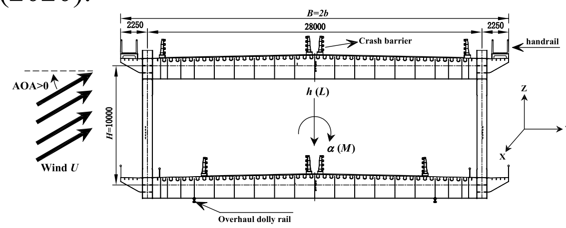


Figure 1. Cross-section of the truss girder (Unit: mm).

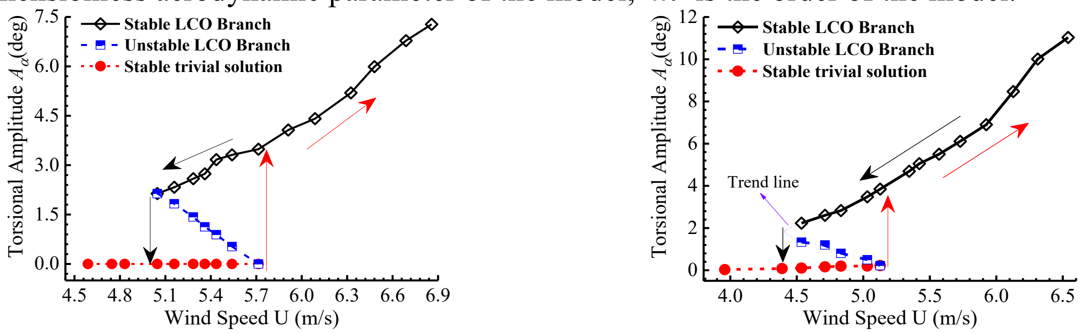
Figure 2 shows the torsional amplitudes of the SDOF and 2DOFs systems. It is observed that the amplitude variation with wind speed was characterized as a typical subcritical Hopf bifurcation, For example, for the SDOF torsional system, attributed to the modal modification effects (i.e., modal properties varying with wind speed) induced by nonlinear self-excited forces, the system performs a subcritical Hopf bifurcation within the wind speed region of [5.0, 5.7]m/s, wherein the unstable LCO decreases with wind speed increasing while the stable LCO increases with wind speed increases. The unstable LCO branch vanishes when wind speed is beyond this region.

3. NONLINEAR SELF EXCITED FORCE MODEL

The SDOF system is used as an example for the investigation. The nonlinear self-excited force model is built as

$$M_{se\alpha}(\alpha,\dot{\alpha}) = \frac{1}{2}\rho U^2(2b^2)\left[\overbrace{A_2(k)\sum_{i=0}^{m}\varepsilon_{i0}(k)|\alpha|^i\frac{b\dot{\alpha}}{U}}^{Damping\ component} + \underbrace{A_3(k)\alpha}_{Stiffness\ component}\right] \tag{1}$$

where ρ is the air density; U is the wind speed; $b = B/2$ is the half-width of the bridge girder; α and $\dot{\alpha}$ are, respectively, the torsional displacement and speed; $A_2(k)$, $A_3(k)$ and $\varepsilon_{i0}(k)$ are the dimensionless aerodynamic parameter of the model; m is the order of the model.



(a) Subcritical Hopf bifurcation of the SDOF system (b) Subcritical Hopf bifurcation of the 2DOFs system

Figure 2. Cross-section of the truss girder (Unit: mm).

4. AERODYNAMIC MECHANISM INTERPRETATION

Figure 3 presents the hysteresis loops for $M_{se\alpha}$ at a wind speed of 5.72m/s. An energy dissipation zone around the equilibrium position, an energy pumping zone at the intermediate amplitude range, and an energy dissipation zone at a large amplitude range can be found. Apparently, ignoring the small dissipation zone around the equilibrium position which can be easily conquered via certain external excitations, the fluid continuously pumps energy into the system. The amplitude will increase when the pumped energy exceeds the dissipative one induced by structural damping. However, the fluid will act as an aerodynamic damper when the amplitude is beyond the pumping zone; the vibration will finally maintain a certain stable LCO wherein the dissipation contribution cancels out the pumping one. However, if the dissipation contribution around the equilibrium position cannot be easily conquered by external excitations, it is convincing that the system will always remain in the still state unless a sufficiently large initial amplitude is applied by which the system can spontaneously increase to another stable LCO. Consequently, two final states exist for the system at one wind speed. Thus, a subcritical Hopf bifurcation emerges in a dynamic system.

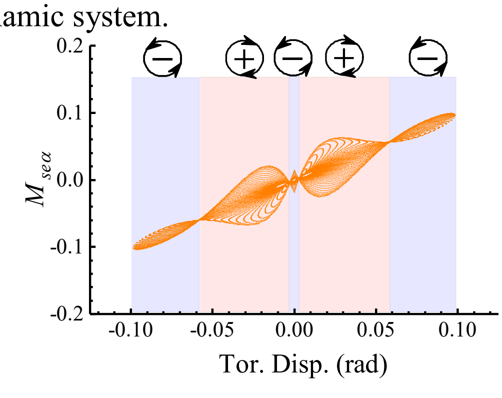
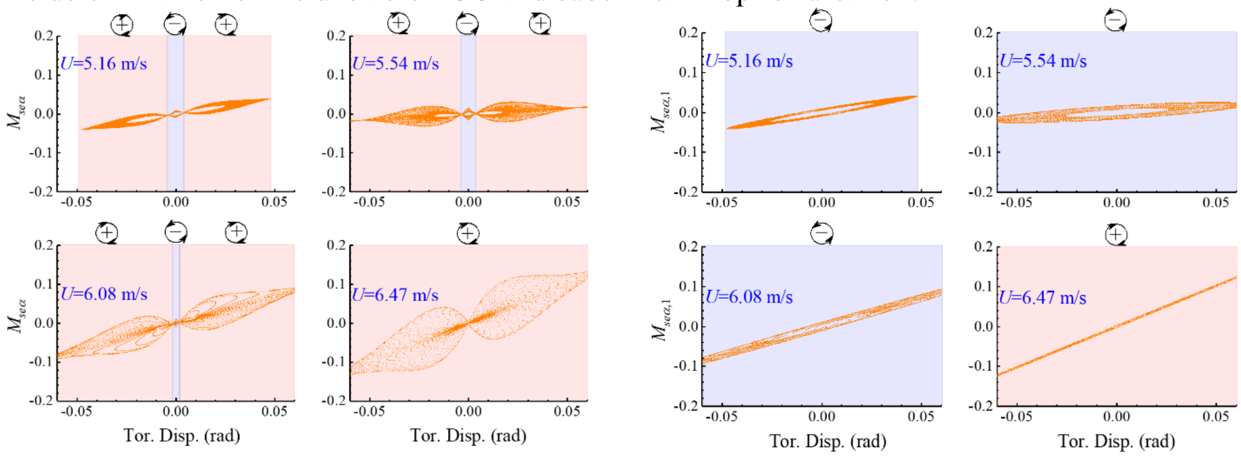


Figure 3. Hysteresis loops of $M_{se\alpha}$ at a wind speed of 5.72m/s.

Figure 4(a) presents the hysteresis loops for $M_{se\alpha}$ at different wind speeds. The dissipation zone around the equilibrium position gradually decreases with increasing wind speed. At a wind speed of 6.47m/s, the dissipation zone vanishes, implying that $M_{se\alpha}$ always pumps energy to the dynamic system even if the initial amplitude is slight. Thus, the subcritical Hopf bifurcation only emerges at a relatively low wind speed range, as shown in Figure 2. Figure 4(b) shows the hysteresis loops of the linear component of $M_{se\alpha}$. One can find that $M_{se\alpha,1}$ dissipates energy in the entire amplitude range when wind speed is less than 6.47m/s. However, at a wind speed of 6.47m/s, the hysteresis loop is clockwise evolved, implying that $M_{se\alpha,1}$ pumps energy to the bridge system. The evolution pattern of energy exchange behavior of $M_{se\alpha,1}$ well explains the change of dissipation zone in the hysteresis loop of $M_{se\alpha}$ varying with wind speed and reveals that the linear component of the nonlinear self-excited-moment plays the most important role in the determination of the unstable LCO and subcritical Hopf bifurcation.



(a) Hysteresis loops of $M_{se\alpha}$ (b) Hysteresis loops of linear component of $M_{se\alpha}$

Figure 4. Hysteresis loops of self-excited force.

5. CONCLUSIONS

This study presented a study on the mechanism of the subcritical Hopf bifurcation of a truss girder. A modified nonlinear self-excited force model was proposed. According to the model, the contribution of nonlinear self-excited forces to the generation of the LCO and the subcritical Hopf bifurcation was quantified. Besides, the real energy feedback mechanism of LCO and the subcritical Hopf bifurcation were investigated through a qualitative explanation in terms of the hysteresis loop of nonlinear self-excited forces.

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REFERENCES

Davenport, A. G., 1968. The dependence of wind loads on meteorological parameters. Proceedings of Proceedings International Research Seminar “Wind effects on buildings and structures", 11-15 Sept. 1967. Ottawa, Canada, 19–82.

Chen, X., Matsumoto, M., Kareem, A. (2000a). Time Domain Flutter and Buffeting Response Analysis of Bridges. Journal of Engineering Mechanics, 126(1),7-16.

Miyata, T. (2003). Historical View of Long-Span Bridge Aerodynamic. Journal of Wind Engineering and Industrial Aerodynamics, 91, 1393-1410.

Amandolese, X., Michelin, S., Choquel, M. (2013). Low Speed Flutter and Limit Cycle Oscillations of a Two-Degree-of-Freedom Flat Plate in a Wind Tunnel. Journal of Fluids and Structures, 43, 244-255.

Wu, B., Wang, Q., Liao, H., et al. (2020b). Hysteresis response of nonlinear flutter of a truss girder: Experimental investigations and theoretical predictions. Computers and Structures, 238, 106267.

Gao, G., Zhu, L., Li, J., et al. (2020). A novel two-degree-of-freedom models of nonlinear self-excited force for coupled flutter instability of bridge decks. Journal of Sound and Vibration, 480, 115406.